THE UNIVERSITY OF ALBERTA

PHOSPHATE ROCK INVENTORY OPTIMIZATION

by

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A THESIS

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ABSTRACT

The objectives of the study were to develop a model that would be useful in controlling Phosphate Rock inventory levels at optimum values under present and future contracts for a variety of economic situations. In addition, the opportunity was taken to assess alternative supply strategies that could have an impact on Inventory Management Policy.

The stated objectives were accomplished by examining historical data associated with the present supply chain and using this data as part of the formulation of a static inventory optimization model. In addition, the static model was tested under conditions of rising prices over time. Lastly, the world demand and supply of Phosphate Rock were reviewed to ascertain the potential for alternative supply strategies.

The study disclosed several important findings which will affect future inventory management policies and the negotiation of future contracts.

1. The risk and opportunities associated with various inventory management strategies can be quantified and as a consequence a practical mathematical model can be used as a tool in inventory management decisions.
2. The costs associated with current inventory management policies are not explicit insofar as a stewardship

function is concerned. The costs associated with current policies are shown to be high thus emphasizing the need for cost accountability and explicit decision rules.

3. Further refinement of the model could be carried out to tie it directly to market forecasts but such a refinement is probably unwarranted due to the relative insensitivity of the model to market changes.

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INTRODUCTION

The Problem

Esso Chemical Canada, a Division of Imperial Oil Enterprises Ltd., owns and operates a fertilizer plant near Redwater, Alberta. One of the raw materials used in the process is Phosphate Rock which is currently imported from Florida, U.S.A. It has been and remains the practice of the fertilizer plant to maintain relatively large inventories of this material to minimize the risk of a "run out" position and consequent plant shut-down and loss of profit. Judgement, knowledge of the security of supply based on past experience, relative inflexibility of the supply chain and cost considerations have dictated the level of supply that was to be maintained.

The impetus for the study arose simply as a result of observance of Phosphate Rock inventory levels at Beamer and at Neptune Terminals. These are summarized in TABLE 17.

Phosphate Rock inventory levels have reached very high levels thus potentially providing a large incentive to develop an optimization model. e.g. A reduction in inventory levels of only 10,000 short tons would result in an annual saving of more than $50,000 in carrying costs based on a cost of capital of 12% and a Phosphate Rock cost of $44/short ton (FOB Beamer).

In order to assess the appropriateness of the current strategy it is necessary to review in detail the past performance of the supply chain, the current and predicted raw material supply, the current and predicted market demand for finished fertilizer products

and the present and future contractual obligations. In addition, a review of supply stewardship and inventory cost accountability may be in order.

The Objectives

The objectives of the thesis are three-fold:

1) To provide a basis for determining optimum inventory levels given the existing contractual arrangements and supply chain at any point in time.

2) To assess the viability of such an inventory management strategy as opposed to alternative strategies given a changing price structure for the raw material i.e. Is the aforementioned strategy appropriate over time?

3) To review alternative raw material supply strategies in light of an inventory optimization policy as well as the attendant considerations of cost, quality and security of supply.

The Reasons

It is appropriate to develop a strategy for optimization of inventory levels under current conditions since the constraints and inflexibilities associated with the present contracts for supply, shipment and handling of the raw material will always exist to some degree with any supply arrangements. Analysis of the present situation is therefore extremely important as such an analysis could have a significant impact on the negotiation of future contracts.

The increase or decrease of Phosphate Rock prices relative to

the cost of capital definitely impact on the appropriateness of any strategy over time. It is therefore necessary to assess the supply/demand situation for Phosphate Rock at any point in time.

At the end of any contractual period the opportunity exists to assess alternatives to the present supply chain. Present contracts end on June 30, 1977 so a review of alternative strategies is particularly appropriate at this time.

Topics of the Paper

Topics of discussion include: Optimization of Inventory Levels with Present Contractual Obligations, Assessment of the Static Optimization Model under Conditions of Changing Price, Alternative Strategies, General Considerations, Summary and Conclusions.

CHAPTER 1

OPTIMIZATION OF INVENTORY LEVELS WITH PRESENT CONTRACTUAL OBLIGATIONS

A. Background Data

1. Supply Chain

Figure 1 depicts the present supply chain. Contracts exist with two Phosphate Rock suppliers, Mobil Chemical Company Division of Mobil Oil Corporation and Swift Agricultural Chemicals Corporation. Both suppliers have mines in Florida, U.S.A. and are responsible for delivery of Phosphate Rock to Tampa, Florida as well as ship loading at Tampa. Movement of Phosphate Rock from the mine sites to Tampa is by independent rail and is a distance of some fifty miles. The contracts between the two companies and Esso Chemical Canada are essentially identical with each company supplying 50% (±5%) of Esso Chemical Canada's requirements up to a maximum of 400,000 short tons in any contract year. (i.e. up to a total maximum of 800,000 short tons)

A contract exists between Star Shipping A/S and Esso Chemical Canada to move Phosphate Rock, by ship, from Tampa, Florida to Vancouver, British Columbia and unload the material at Neptune Terminals. No minimum amount of Phosphate Rock movement is stipulated but a maximum of 1,008,000 short tons is allowable in any contract year. Each ship carries approximately 28,000 short tons.

A contract exists between Neptune Phosphate Terminals Limited and Esso Chemical Canada to receive and store Phosphate Rock received from Star Shipping A/S and load Canadian National Railway

cars destined for Esso Chemical Canada's fertilizer plant upon request. Silos having a storage capacity of 60,000 short tons exist at Neptune Terminals. Neptune Terminals is prepared to receive and handle up to 1,200 short tons/hour, twenty-four hours per day exclusive of Sundays and holidays. Esso Chemical Canada shall supply a minimum of 190,000 short tons to Neptune Terminals in any contract year or be assessed penalties as agreed to in the contract.

A contract exists between Canadian National Railways and Esso Chemical Canada to move Phosphate Rock, by rail, in unit trains of 85 to 101 cars, from Neptune Terminals at Vancouver, British Columbia to Esso Chemical Canada's fertilizer plant at Beamer, Alberta. Canadian National Railways guarantees to move a minimum of 816,000 short tons in any contract year. Esso Chemical Canada shall supply a minimum of 714,000 short tons to Canadian National Railways in any contract year or be assessed penalties as agreed to in the contract.

In summary, two suppliers supply Phosphate Rock from mines in Florida that is transported by independent rail to a port at Tampa, Florida. The Phosphate Rock is then moved by ship via the Panama Canal to Vancouver, British Columbia and subsequently moved by rail to Esso Chemical Canada's fertilizer plant at Beamer, Alberta.

2. Significant Contract Details

a) Contracts with Mobil and Swift [1, 2]

- Each contract is a five year contract and ends on June 30, 1977.

- Each supplier will supply 50% (±5%) of Esso Chemical

Canada's requirements up to a maximum of 400,000 short tons in any contract year (July 1 to June 30). No minimum amounts are specified although it is stated that Esso Chemical Canada will generally require between 225,000 short tons and 250,000 short tons from each supplier per contract year.

- Normal "Force Majeure" clauses exist to protect both parties from penalties resulting from natural and unnatural calamities.

b) Contract with Star Shipping [3]

- The contract is a two year contract that ends on June 30, 1977.

- The contract states that the normal amount of Phosphate Rock to be moved in any contract year (July 1 to June 30) will be between 784,000 and 1,008,000 short tons. The maximum quantity that may be shipped is 1,008,000 short tons in any contract year. No minimum amount is specified.

- The main feature of this contract is the relatively elaborate procedure for scheduling ship movement. The basic requirement that is significant to this study is as follows: A schedule is prepared by Esso Chemical Canada, at least three months in advance of any contract year, for the coming contract year. Star Shipping, upon acceptance of the schedule for the contract year, agrees to supply ships on the commencement dates (±10 days). However, Esso Chemical Canada has the right to change any commencement date by

giving at least 45 days notice to Star Shipping.

- In the event that the Panama Canal passage is restricted for any means whatsoever either party will have the right to suspend the agreement. In the event that neither party elects to suspend the agreement vessels shall proceed to the discharge port by the most economical alternative route. Additional costs incurred will be borne by Esso Chemical Canada at rates specified in the contract.

- Vessel loading will be at minimum rates of 16,800 short tons per 24 hour day (Sundays and holidays included excepting Christmas, New Years, July 4th and Labour Day) commencing on the commencement date specified by 6 days of notice. Swift or Mobil will pay any penalties for any delays not associated with weather conditions.

- Normal "Force Majeure" clauses exist to protect both parties from penalties resulting from natural and unnatural calamities.

c) <u>Contract with Neptune Phosphate Terminals</u> [4]

- This contract is a fifteen year contract that ends on June 1, 1984.

- Esso Chemical Canada shall supply to Neptune Terminals a minimum of 190,000 short tons in any accounting period until a total supply of 2,850,000 short tons has been achieved or be subject to penalties described in the contract.

- Vessel unloading rates will be maintained up to 1,200 tons

per hour or demurrage costs will be borne by Neptune Terminals. (Sundays and holidays excluded)

- Neptune Terminals shall provide storage for up to 60,000 short tons of Phosphate Rock and operate the dehumidifier in order to maintain the Phosphate Rock in a dry condition.

- Neptune Terminals shall operate its facilities as requested to accomplish the entire operation of loading, weighing and assembling 50 railway cars within the period of one 8-hour shift.

- Esso Chemical Canada can at any time suspend this agreement by giving three month's notice in writing to Neptune Terminals provided the contract tonnage obligation of 2,850,000 short tons has been surpassed.

- A normal "Force Majeure" clause exists to protect both parties from penalties resulting from natural and unnatural calamities.

d) Contract with Canadian National Railways [5]

- This contract became effective on November 15, 1974 and ends on June 30, 1977.

- Esso Chemical Canada, through its agent Neptune Terminals must load 85 car trainloads in 24 hours or be subject to demurrage.

- Canadian National Railways guarantees a supply of eight unit trains of 85 cars each (100 short ton capacity) per month i.e. 816,000 short tons per year. This total can be

increased by negotiation by 100,000 short tons per year.

- Esso Chemical Canada guarantees a minimum supply of Phosphate Rock of 51,000 short tons per month and 714,000 short tons per year.
- A normal "Force Majeure" clause exists to protect both parties from penalties resulting from natural and unnatural calamities.

e) Contract Summary and Discussion

Phosphate Rock requirements at the fertilizer plant will be between 400,000 and 800,000 short tons, the variation being largely dependent upon fertilizer market requirements. As a result it is apparent that all contracts allow for the maximum requirements of Esso Chemical Canada (TABLE 1). The requirement of 2,850,000 short tons with Neptune Terminals has been surpassed so is no longer a consideration. Therefore only the Canadian National Railway Contract is of any concern in carrying out an inventory management policy. i.e. In times of a soft market it becomes necessary under the Canadian National Railway Contract to either build inventory or pay a penalty of $0.85 per short ton on contract underrages.

Another area of interest is the degree of inflexibility that exists with the contracts as a whole but particularly with the Star Shipping and Canadian National Railway Contracts. In the case of the former a schedule for the

next contract year must be prepared and agreed to three months in advance of the contract year. Changes can then be made by Esso Chemical Canada to any voyage if 45 days notice is provided. However, desired schedule changes do not necessarily occur automatically. i.e. Requests for additional tonnage would be dependent upon the availability of ships. Also, the intent of the schedule is to provide as even a flow of Phosphate Rock as possible. Disruptions (i.e. cancellations) could result in lost revenue to Star Shipping and be detrimental to future requests.

In the case of the Canadian National Railway Contract minimum rates are actually specified on a monthly as well as an annual basis. This restriction makes it difficult to adjust inventories downward rapidly without incurring penalties.

The last area of interest is the fact that all contracts end or can be terminated on June 30, 1977.

In summary, the Contracts as they stand do not negate the possibility of Inventory Management. However, the system is relatively inflexible in any short term and thus places a fairly significant onus on marketing forecasts for fertilizer products.

B. Assessment of Historical Data

As a first step in providing a data base for any mathematical model it was felt to be necessary to quantify the risk associated with Phosphate Rock supply. The obvious data for this type of analysis is historical data. Two questions therefore had to be answered:

- Was data available?
- If the data was available could it actually be considered to be suitable for predicting the future?

1. Data Availability

Three types of historical data were found to be available and thought to be appropriate for this analysis.

- Ship scheduled arrival time at Tampa, Florida vs. planned arrival time.

The contract calls for an agreed to arrival date ±10 days. The ability of Star Shipping to meet the planned dates was deemed to be important. This information is summarized in TABLE 2.

- Elapsed time between the commencement of ship loading in Tampa, Florida and the completion of ship unloading in Vancouver, B.C.

This information tends to capture a number of logistics operations but additional information exists to identify the cause of most delays. This information is summarized in TABLE 3.

- Elapsed time between loading commencement of railway cars at Vancouver, B.C. and the completion of car unloading at Beamer, Alberta.

This area was separated since the block movement was different from the previous movement (i.e. 28,000 short ton ships vs. 8,500 short ton unit trains) and since the commencement of railway car loading was somewhat discretionary i.e. Based upon inventory considerations in the Vancouver silos and at Beamer, Alberta and not necessarily completely on the arrival time of ships. This information is summarized in TABLE 4.

2. Data Suitability

In order to assess the suitability of the data in TABLES 2, 3 and 4 it was necessary to assess the causes of abnormal deviations in the data and also to review each leg of the transportation chain to determine events that could in future periods have a significant abnormal effect on trip durations. The following observations are of interest:

a) The data spans the entire history from December, 1974 to mid 1976. This period includes two Christmas/New Year periods - periods often considered critical as a result of labor problems. Also, one major strike at Neptune Terminals is included in the data and a period of shortage of supply of Phosphate Rock.

b) An examination of TABLES 2, 3 and 4 indicates that the

greatest degree of uncertainty is associated with the ship arrival times at Tampa relative to the planned arrival times. A review of the data for voyages 100 to 112 revealed that this period

- included a strike at Neptune Terminals
- was a period when Phosphate Rock was in short supply
- was a time of ship congestion at Tampa.

Ship schedules were thus modified intentionally to avoid high demurrage costs.

A further examination of TABLES 2, 3 and 4 indicates the low degree of variability in the longest leg of the trip. i.e. Tampa to Vancouver including ship loading and unloading. A more detailed review of this result, data not included, indicates that essentially all the variability was associated with ship unloading at Vancouver.

c) A review of the data and union contracts etc. for each leg of the trip revealed the following:

- The Seaboard Coast Line is the railway handling Phosphate Rock from the two Florida mines to the ships at Tampa. The railroads' responsibilities include railroading, weighing and terminalling. No alternative rail line from the mines to Tampa exists.

Three terminals are located at Tampa. Rockport Terminal is a public terminal and the terminal through which E C C Phosphate Rock is moved. Two unions exist

at this terminal, the International Longshoremans Association (ILA) and the International Longshoremans Association Terminal (ILAT). Contracts with each union have typically been three year contracts, the current ones expiring September 30, 1977. Strikes lasting as long as 90 days have occurred.

Eastern Associated Terminal is a private terminal not used by the two Phosphate Rock suppliers. It uses non-union labor.

International Minerals Corporation Terminal is not used for Phosphate Rock.

Star Shipping employees are non-unionized so no potential strike problems exist. Steaming time from Tampa to Vancouver has been relatively invariant. Even if problems occurred at the Panama Canal it would be to the mutual benefit of both parties to agree to an alternative route. The additional steaming time would not be significant given even minimum levels of inventory.

The agreement with Star Shipping is a backhaul arrangement and present demand for vessels is slack and predicted to remain so for some time. Star Shipping, therefore, has seldom had a problem meeting planned arrival dates at Tampa and will have no additional problems in the near future. The numbers determined in

TABLE 2 thus do not represent real delays in arrival at Tampa but normally represent requests from E C C to delay the arrival times of vessels.

- Stevedores and longshoremen at Neptune Terminals are unionized and both have annual contracts that expire on January 1. These unions are strong and have a history of strikes but it is worthwhile to recognize that wages paid to these groups are a small percentage of the value of any cargo and that the economy of Canada would suffer tremendously with any extended strike and thus it is unlikely that a strike would be allowed to continue for a lengthy period of time.

- Canadian National Railways can and have had business interruptions due to slides and things of that nature but these events occur very seldom, can normally be remedied in fairly short order (a few days) and alternative railroading via the Canadian Pacific Railway will always provide some alleviation.

The C.N.R. has to deal with some fifty unions that could disrupt business but as with Neptune Terminals it is unlikely that the Federal Government would allow any strike to carry on for an extended period of time.

Car availability is always a factor but this is accounted for in the statistical data and as well the contract for unit trains has some advantage over

normal railroading.

In summary, it is apparent that the data collected can be separated into two different classifications. First, voyages 100 to 112 inclusive represent a period of turmoil caused by three factors - Phosphate Rock supply problems, a strike at Neptune Terminals and ship congestion at Tampa. Second, voyages 113 to 132 inclusive (excluding voyage 131) represent what could be considered to be "normal" conditions.

Further assessment will therefore be concerned with a rigorous analysis of the second classification to determine optimum inventory levels under "normal" conditions. An overall strategy is then proposed that is cognizant of the realities of the former classification.

3. Data Assessment for "Normal" Conditions

The assessment of the data falls into two areas; firstly, the specification of the probability distribution from which the sample was drawn and secondly, the quantification of the variability of the data.

a) Goodness of Fit Test [6]

A review of the data and a knowledge of the actual situation indicated that the data was probably normally distributed. The following testing procedure was employed:

Ho: Sample data (trip durations) are normally distributed with mean μ = 38.74 days and standard deviation σ = 6.51 (see Appendix I and TABLE 5 for calculations of

μ and σ.

Hi: Sample data (trip durations) come from some other distribution

Test Statistic: $$\chi^2 = \sum \frac{(f_a - f_e)^2}{f_e}$$

Decision Rules: Accept Ho if $\chi^2 \leq \chi^2_\alpha$

Reject Ho if $\chi^2 > \chi^2_\alpha$

In order to test the data it was grouped into blocks of ten days. Figure 2 summarizes the data in the form of a histogram. TABLE 7 summarizes the details of the calculations employed in the testing procedure. From these calculations it is determined that $\chi^2 = 3.015$ which means that the null hypothesis is accepted at a level of significance of 0.01 ($\chi^2_{0.01} = 16.812$).

b) <u>Confidence Intervals</u>

TABLE 8 summarizes the variability data for the trip durations from Tampa to Beamer. The important element to examine insofar as this analysis is concerned is the positive deviation from the mean. i.e. Ships are scheduled on a regular basis and on average an elapsed time of 38.74 days from the planned arrival date of the ship at Tampa until the unit trains are completely unloaded at Beamer. Therefore, the positive deviation from this mean value is the value that dictates the level of inventory that should be maintained at Beamer.

c) Key Points Relative to Data Assessment

A number of points are worthy of further comment as they will be important in the subsequent analysis.

- The analysis of "Normal" conditions included all three voyage elements originally identified. i.e. The trip labelled "Tampa - Beamer" is the time of travel based on the original planned arrival time of a ship at Tampa rather than the actual arrival time of a ship at Tampa. It is important to capture the arrival variance at Tampa because it is shown to be significant and because it allows the subsequent model that is developed to be used on a planned basis.

- The mean trip time from Tampa to Beamer is calculated to be 38.74 days. This figure obviously does not compare very well with the figure of 27 days obtained by summing the average time in FIGURE 1. (2 + 17 + 2 + 1 + 4 + 1). A number of reasons exist to explain this discrepancy. First, the analysis used captures the time difference between actual and planned arrival times of ships at Tampa. Second, the numbers used in FIGURE 1 are figures based on normally expected good times once any particular operation is initiated. Third, and most important, is that the last leg of the trip from Vancouver to Beamer is shown to require 5 days (4 days + 1 day for unloading) in FIGURE 1. However, this is the time required to move only

one unit train. Nearly four unit trains are required to transport the contents of any one ship from Vancouver to Beamer and this was the basis for the "Pseudo Voyage" originally identified in TABLE 4.

- It is important to recognize that the subsequent analysis (next section) is in no way dependent on the mean time. The variability of the trips is the important factor since the variability is essentially the measure of risk associated with any strategy. In this analysis the standard deviation is used. The three elements chosen previously were chosen in a purposeful way so that the total variation would be captured by any model that was developed.

- It can be argued that with the last leg of the trip (Vancouver to Beamer) we need only be concerned with the time required for the first train to arrive at Beamer as this is what will determine if a runout position will occur at Beamer. However, an analysis on this basis would be unacceptable for a number of reasons. First, as stated before only the trip variance is important and not the mean of the trip. Thus an analysis on the basis of single train movements would be concerned with the variance associated with those trips rather than the mean times. Second, using single trip times as a basis would be inappropriate since the variance associated with unit

train scheduling would not be captured. Third, it is not possible to associate one particular unit train with a ship's arrival due to the surge capacity existing at Neptune Terminals.

The analysis used here of identifying a "Pseudo Voyage" is much more appropriate since the movement of four unit trains can more readily be associated with one particular ship and since the variances associated with loading, travelling, unloading and scheduling are all captured in a single number.

- A review of the data in TABLE 6 is interesting since for independent random variables the following relationships hold:[7]

(1) $\text{Mean}(X_1+X_2+---+X_n)=\text{Mean}(X_1)+\text{Mean}(X_2)+---+\text{Mean}(X_n)$

(2) $\text{Variance}(X_1+X_2+---+X_n)=\text{Variance}(X_1)+\text{Variance}(X_2)+---+\text{Variance}(X_n)$

If the three elements were independent the mean and standard deviation for the sum (Tampa - Beamer) would be 38.74 and 8.32 respectively.

The lower value of 6.51 for the standard deviation indicates a degree of dependency among the three elements. This essentially verifies a fact known from observation. i.e. If a delay occurs on any leg of the trip additional

effort will be made to reduce the time associated with any subsequent leg of a trip.

C. Inventory Model for "Normal" Conditions

1. Basis for Model

Inventory theory[10] is concerned with two different cases; the deterministic cases in which all demand and supply rates are fixed at constant values and the probabalistic cases in which at least one component is variable in a probabilistic fashion. The most general case occurs when both a probabilistic supply function and a probabilistic demand function exist.

Answers to two main questions are generally sought; when to order and how much to order. Two main issues can exist if inventories are completely depleted; fulfillment of orders can be delayed giving rise to the "Back Order Case" or sales can be lost forever giving rise to the "Lost Sales Case".

Competing costs involved in any analysis can include ordering costs that are generally related to the size of the order, carrying costs that are related to the lost opportunity cost, warehousing costs etc. and lost sales costs that are related to the loss of profitability and goodwill.

With this general background it is possible to turn to the specific case at hand. FIGURE 3 depicts the type of inventory situation that is being analyzed. The following points serve to clarify the bases for the model and also identify its limitations.

- The model is not typical of the classical inventory model due to the relatively long mean delivery time compared to the consumption rate. i.e. line r^* indicates the reorder point

and T^* the mean travel time from time of order to delivery at Beamer, Alberta (μ = 38.74 days) whereas T is the time required to completely consume the contents of one ship (10 to 28 days dependent on the rate chosen for analysis).

- The order quantity, Q, is fixed at the value of one ship cargo by contract. Q is taken as a constant at 28,000 short tons. The small amount of variability in the sizes of ship cargoes will not significantly affect the analysis. Also, since the quantity, Q, is constant the ordering cost is also constant and does not affect subsequent analysis in the normal fashion. i.e. It is included as part of the purchase cost of inventory and thus has an impact on carrying cost.
- A cycle is deemed to occupy time period T. i.e. The number of cycles per year are equivalent to the number of ships per year.
- The supply function is probabilistic as shown in the previous section. i.e. It is a normally distributed function with mean μ = 38.74 and standard deviation σ = 6.51. Thus, on average the supply of Phosphate Rock will be replenished 38.74 days after the order date has been fixed. Another s/u days (where s is the safety stock and u is the consumption rate) can pass before the inventory is completely depleted. The probability of a ship arriving after μ + s/u days does exist and is the probability that a runout of Phosphate Rock will occur in any cycle.

- The rise in inventory at Beamer is shown as a vertical rise in FIGURE 3 of 28,000 short tons. This is an over-simplification since:

  - Unloading at Beamer cannot occur instantaneously.
  - Arrival of Phosphate Rock at Beamer is in 8,500 short ton lots rather than 28,000 short ton lots.

By using this simplification a simple formula can, however, be derived for the average inventory I. The actual figure will be less by 10,000 short tons at most. (28,000 $\div$ 2 minus 8,500 $\div$ 2) and thus merely adds a degree of conservatism to the model. However, it is likely that use of the model would not be at all impaired by this simplification as it would be easier to steward to a level of S rather than to a level of I.

- The consumption rate or demand rate is shown as being linear. The actual consumption rate is probabilistic (i.e. A function of demand for finished product) but can be taken as a constant for purposes of analysis. This is a result of the seasonal demand for finished product that necessitates the carrying of large finished product inventories. Thus the demand in any quarter is simply the planned production rate for that quarter that is dictated by market predictions and is a constant or at least can be averaged for any particular quarter.

The demand on a daily basis will naturally be variable due

to both scheduled and unscheduled shutdowns but the average demand remains constant until new market forecasts are prepared.

Model generalization sufficient to allow recalculation of optimum inventory levels in light of changed market demands is all that is necessary to accomodate the "probabalistic" demand function.

- The model can be described neither as a pure "Back Order Case" nor a pure "Lost Sales Case" due to the seasonal demand for finished product and also due to the cyclical demand for finished product over a longer time frame (6 to 10 years) i.e. When demand is such that all the product produced by the plant throughout the year can be sold and sold at a profit then any loss of production equates to the true "Lost Sales Case". However, when demand for finished products is less than maximum plant throughput then any loss of production, if not excessive, can be made up at a later time by running at higher rates.

Costs incurred in the two different modes of operation will thus be different. In the case of maximum demand the costs will be large while in the case of low demand the costs will be small. It is interesting to note that the costs in the latter case are not necessarily zero since unscheduled shutdowns can have numerous unplanned variable costs associated with them including poor energy and manpower utilization plus

the potential for mechanical damage that is more prevalent under shutdown and startup conditions.

In order to accomodate the two extremes that can occur the model was generalized by determining optimum inventory levels under a variety of shutdown costs. These numbers are discussed further after the development of the mathematical model.

The basis for the model can thus be summarized as follows: Order quantities and ordering costs can be treated as constants. The supply function is probabilistic and shown to be normally distributed. The demand function is probabilistic but can be treated as a constant. The circumstances cannot be classed as a "Back Order Case" nor a "Lost Sales Case". A generalized solution can thus be obtained by treating u (the demand for Phosphate Rock) and k (the lost production cost) as constants during any quarter but variable in the longer term.

2. Model Development

Two costs are considered to be important in the analysis. They are the Carrying Cost of Inventory (normally referred to as the Holding Cost) and Lost Production Cost (normally referred to as the Stockout Cost.)

(1) Total Cost(TC)=Carrying Cost of Inventory(CC) + Lost Production Cost (LPC)

The optimum inventory level, of course, will occur at a level that minimizes the value of the Total Cost Function. Other costs exist but are ignored for various reasons:

- The C.N.R. contract calls for a penalty of $0.85 per short ton for less than contract tonnages (51,000 short tons per month and 714,000 short tons per year). This contract ends June 30, 1977 and must be renegotiated as it is far too restrictive. i.e. Such a penalty virtually destroys any opportunity to optimize inventory levels during times of low demand for finished products.

  This penalty was thus ignored as it is assumed that renegotiation will have to take place.

- Ordering costs as mentioned previously are constant and thus reflected in the value of inventory and the attendent Carrying Cost.

- Costs associated with wind losses and handling are both ignored for simplicity. Inclusion would only tend to lower the calculated Optimum Inventory Levels and thus exclusion adds a degree of conservatism to the results. i.e. Both costs increase with increasing levels of inventory.

The quantification of the two costs being assessed is as follows:

(2) $CC = I\ C_o r$

The equation simply says that the Carrying Cost of Inventory is

equivalent to the Lost Opportunity Cost for other ventures of comparable risk.

Using the Cost of Capital (r) to evaluate the Carrying Cost assumes that the corporation is not "cash poor". i.e. It assumes that other opportunities with comparable risk and higher returns are not being passed by due to lack of funds.[11]

$$(3)\ \text{LPC} = \frac{365u}{Q} \int_{\mu + \frac{s}{u}}^{\infty} k \left[x - \left(\mu + \frac{s}{u} \right) \right] f(x)\,dx$$

This equation[12] in words says that the Lost Production Cost is equal to the probability of running out of Phosphate Rock in any given cycle times the cost of running out of Phosphate Rock times the number of cycles in a year. i.e. Equation (3) is composed of the three elements described by equations (4), (5) and (6) that follow.

(4) Number of Cycles per year $= \frac{365u}{Q}$

(5) Probability of the Cargo arriving on the x th day $= f(x)$

The probability function in this case has been shown previously to be Normal.

(6) Cost of Running Out of Rock $= k \left[x - \left(\mu + \frac{s}{u} \right) \right]$

Beyond day $\mu + s/u$ a probability of runout exists; hence the

reason for the integral limits. Therefore, as well, for $x > \mu + s/u$ the expression in brackets gives the number of days for which a probability of stockout does exist. The constant k, as previously discussed, represents the daily cost of a runout situation.

Inspection of FIGURE 3 yields the following expression for the average inventory, I.

$$(7)\ I = S + \frac{Q}{2}$$

This expression is not completely valid for two reasons:

- As discussed previously the inventory at Beamer does not increase instantaneously by a value of Q. However, this simply adds a degree conservatism of between zero and 10,000 short tons to the final determined optimum inventory level.
- Since the model plans for the inventory level to fall below zero at times the value determined by equation (7) will be lower than the actual average level. This offsets to a degree the previously discussed error, however, the adjustment to be made is small and thus not worth considering.

e.g. For values of S and u of 36,000 short tons and 2,000 short tons/day respectively (This value of S is shown to be near optimum by subsequent analysis and the value of u of 2,000 is near maximum) the following

is true:

$$\mu + \frac{s}{u} = 38.74 + 18 = 56.74$$

$$z = \frac{x-\mu}{\sigma} = \frac{18}{6.51} = 2.76$$

$$\therefore P(x > 56.74) = .0029$$

This probability is so small that it has no significant effect on the average inventory level, I.

Substitution of equations (2), (3) and (7) in equation (1) yields the following:

$$(8)\ T.C. = SC_o r + \frac{Q}{2} C_o r + \frac{365u}{Q} \int_{\mu+\frac{s}{u}}^{\infty} k \left[x - \left(\mu + \frac{s}{u}\right)\right] f(x)\,dx$$

Differentiating with respect to S and equating the subsequent expression to zero allows an expression to be found for the optimum value of S and thus the optimum value of I.[13] The differentiation of the integral is accomplished by making use of Liebniz' rule.

$$(9)\ \frac{d(T.C.)}{ds} = 0 = C_o r - \frac{365k}{Q} \int_{\mu+\frac{s}{u}}^{\infty} f(x)\,dx$$

Now the integral of the density function is simply equal to the cumulative probability distribution function, F.

$$(10)\ 0 = C_o r - \frac{365k}{Q} \left[F(\infty) - F\left(\mu + \frac{s}{u}\right)\right]$$

and $F(\infty) = 1.0$

$$(11) \quad G\left(\mu + \frac{S}{u}\right) = 1 - \frac{C_o r Q}{2 A R k}$$

Substitution of known values in the right hand side of this expression determines a value which can be used to obtain a value of z from Standard Normal Curve Tables.

Now[14] $$(12) \quad z = \frac{x - \mu}{\sigma}$$

and $$(13) \quad x = \mu + \frac{S}{u}$$

Substituting equation (13) in equation (12) yields:

$$(14) \quad S = u \sigma z$$

Substituting equation (7) in equation (14) yields:

$$(15) \quad I = \frac{Q}{2} + \mu \sigma z$$

Where S and I in equations (11),(14) and (15) are the optimum values of S and I.

For purposes of reference and analysis it was felt to be appropriate to use the above equations to prepare a generalized solution. This was accomplished by using a number of values for the parameters in the derived expressions to produce a plot of optimum inventory levels versus plant demand rate at various lost production penalties. C_O, r and Q are relatively invariant in the short term and were thus held constant. u and k were varied to obtain the desired results.

As discussed previously the cost of running out of Phosphate Rock (k) can vary from a very large value when the full plant capacity is demanded to a very small value when the demand for finished product is low (i.e. less than plant capacity). The former costs can be simply determined by assessing the lost contribution to earnings that results from a plant shutdown. (Product Netback minus Variable Cost). This cost will vary dependent on the market block in which the incremental tonnage is sold and the profitability associated with fertilizer sales at any point in time. The latter costs, as previously discussed, are associated with the inability to avoid variable costs with unplanned shutdowns and the greater potential for mechanical damage associated with shutdown and startup situations.

In order to determine a range of Lost Production Costs the current margins in the various market blocks based on a weighted product mix using the Spring Financial Forecast for

1977 were determined. Based on this analysis losses of \$10,000/day, \$50,000/day and \$80,000/day were assessed since they would essentially encompass all demand and market block considerations.

The plant demand (u) will normally be in the 2,000 short ton/day range. Plant demands of 1,000, 2,000 and 3,000 short tons/day were used to develop the aforementioned graphs and encompass all plant demands of interest. i.e. The zero demand case is not of any interest.

The results of the calculations are summarized in FIGURE 4. The following example serves to illustrate the methodology.

e.g. C_o = \$44/short ton
r = 10%
Q = 28,000 short tons
u = 3,000 short tons/day
k = \$80,000/day

Substitution in equation (11) yields:

$$F\left(\mu + \frac{s}{u}\right) = 1 - \frac{(44)\ (0.10)\ (28,000)}{(365)\ (80,000)} = 0.9957$$

From TABLES $Z = 2.635$

Substitution in equation (14) yields:

$$S = (3,000)\ (6.514)\ (2.635) = 51,500 \text{ short tons}$$

Substitution in equation (7) yields:

$$I = 51,500 + \frac{28,000}{2} = 65,500 \text{ short tons.}$$

D. Inventory Management for "Normal" and "Abnormal" Conditions

Two main "abnormal" conditions can potentially occur. First, the suppliers of the Phosphate Rock may be reluctant to meet their commitments to Esso Chemical Canada if greater opportunities for profit exist by supplying other users who are prepared to pay higher prices. Second, strikes can occur.

Neither issue is dealt with in a rigorous mathematical fashion. However, it is clear that a significant incentive exists to lower inventory levels from the present level of 150,000 short tons to a level nearer 50,000 short tons - an incentive in the order of $500,000 per year. Therefore, a strategy that is cognizant of both the optimum levels of inventory under "normal" conditions and "abnormal" conditions that can occur is appropriate. The following is proposed:

- Inventory levels normally be maintained at a level dictated by the uppermost curve in FIGURE 4 at the point of maximum plant capability. Under present conditions an inventory level between 45,000 and 55,000 short tons would be appropriate.

  Such a strategy is conservative for "normal" conditions due to the aforementioned model simplifications that added a degree of conservatism, does not take credit for Phosphate Rock inventory at Vancouver and handles the potentially most damaging economic case (high lost production cost and high plant rate).

- The first "abnormal" condition (short supply) be handled simply by monitoring the supply situation. This would include

being aware of suppliers inventory situations and the demand/ supply picture for Phosphate Rock. Times of tight supply would coincide with profitable times for fertilizer products as well (i.e. times of high demand) and thus dictate that Phosphate Rock inventory levels at Beamer be increased.

Historical data from TABLE 2 indicates a general shift of twenty-three days for all ships during the critical period. Inventory management policy for normal conditions allows approximately fifteen days of delay of ships prior to a run-out condition occurring. Increasing inventory levels by the capacity of one ship would thus have virtually assured the avoidance of a runout condition.

Thus it is probably appropriate to increase inventory levels at Beamer to 80,000 to 100,000 short tons prior to the commencement of a tight supply situation.

- The second "abnormal" condition (strikes) be handled by building inventory in the three month period prior to the expiration date of union contracts at Tampa or Vancouver. The level would be reduced to "normal" levels when it became clear that a settlement had been reached at the port in question.

Such a strategy involves building inventory annually in the October/November period and reducing inventory in the January/February/March period to accomodate the Vancouver contracts. In addition the Tampa contracts would have to be accomodated every three years.

Again, a review of past data and the potential for the involvement of the Federal Government in any sustained strike would suggest an increase of 30,000 to 50,000 short tons (one or two ships) would be an appropriate increase.

F. Summary and Conclusions

1. A practical inventory model has been developed that can be used as a tool for Inventory Management under "Normal" conditions. This model, although developed in a very conservative fashion, indicates that the maximum optimum inventory level at Beamer is less than 55,000 short tons. Under conditions of low demand for finished product the optimum level is much less.

2. A strategy of raising and lowering inventory levels from theoretical optimum levels prior to and after, respectively, union contract termination dates and the onset of "tight" Phosphate Rock supply periods can be employed to deal with "Abnormal" conditions. Such a strategy would increase the yearly average inventory to a level of 65,000 short tons. The adoption of such a strategy would reduce current inventory levels by nearly 90,000 short tons and thus yield an annual saving of $500,000 based on a Phosphate Rock cost of $44 per short ton (FOB Beamer) and a Cost of Capital of 12%.

3. The current contract with the C.N.R. must be renegotiated if the above savings are to be realized during "soft" market conditions for finished fertilizers.

CHAPTER 2

ASSESSMENT OF THE STATIC OPTIMIZATION MODEL UNDER CONDITIONS OF CHANGING PRICE

A. Background

During the course of reviewing the world supply of Phosphate Rock in preparation for contract negotiations in 1977 it became apparent that the cost of Phosphate Rock was going to increase significantly over the next few years.[15, 16, 17] (See TABLE 9) As a result, the question arose as to whether there was an economic incentive to increase the take of Phosphate Rock to the maximum allowed by the contracts with Mobil and Swift. Testing of this approach essentially tests the previously developed static optimization model under conditions of rising cost for Phosphate Rock.

B. Alternative Strategies

Two alternative strategies were considered to test the static model and are depicted in FIGURE 5. The first strategy simply involves the continuous maintenance of an average Phosphate Rock inventory level at Beamer of 50,000 short tons. This figure, based on the previous optimums determined in Chapter 1, is not optimum for all usage rates depicted in FIGURE 5. At $50,000/day the actual optimums range from 25,000 short tons to 60,000 short tons and are thus less than 50,000 short tons on average. As a result, the use of 50,000 short tons biases the comparison of the two strategies in favour of the second strategy thus providing a rigorous test for the previously developed optimum inventory level model.

The second alternative strategy involves increasing the Phosphate Rock inventory level at Beamer from 50,000 short tons to 550,000 short tons over a one year period and then consuming this Phosphate Rock until an inventory level of 50,000 short tons is reached at some later time. Points A, B, C and D indicate the time required to reach the 50,000 short ton inventory level at respective plant consumption rates of 685, 912, 1370 and 2732 short tons/day. All are based on no Phosphate Rock being purchased during the run down periods.

A figure of 550,000 short tons is chosen for the second strategy so that the comparison will be biased in favour of the second strategy and thus provide a rigorous test for the previously developed optimum inventory level model. i.e. If the second strategy

is sound economically then the most rapid buildup that is possible will yield the highest return. A buildup to 550,000 short tons in a one year period thus enhances the second strategy since:

1. All the Phosphate Rock will be purchased at the minimum possible cost, C_1.

2. From a practical point of view it would be very difficult to build inventory levels by 500,000 short tons in one year due to the lack of storage capacity and the difficulty in scheduling such a large movement.

The dashed curve in FIGURE 4 depicts the rising cost of Phosphate Rock during the period under consideration. For simplicity of analysis the increments during the period under consideration are considered to be equal.

C. The Economic Model

In order to compare the two strategies the Present Value of the costs associated with each strategy are determined and compared. Three costs are considered to be significant - the carrying cost associated with inventory, the actual purchase cost of the Phosphate Rock and the cost associated with lost profit due to a Phosphate Rock run out situation.

The factors of greatest importance in an analysis such as this are the opportunity cost associated with the carrying cost of inventory and the discount rate to be used in determining the present value of a future stream of costs. The opportunity cost was previously discussed and was shown to be the Cost of Capital. The discount rate to be used must be such that net present costs having a value of zero will cause the market value of the firm to remain unchanged. Therefore, the discount rate must also be the Cost of Capital.[11]

Formulae for determining the present values of Carrying Cost and Purchase cost can be determined by direct inspection of FIGURE 5. The derivation of a Carrying Cost formula in each case simply involve determining the average inventory level maintained in each period, multiplying it by the cost of the inventory and applying the Cost of Capital to quantify the opportunity cost and to discount the period cost to the present. The present values of the costs of each period are then summed to determine the total present value for a particular strategy and consumption rate.

e.g. $u = 1370$ short tons/day
Second period (Mid 1977 to Mid 1978)

$$\text{Average Inventory} = \frac{I^* + I}{2}$$

$$\text{Cost of Inventory} = C_1$$

$$\text{Present Value for Period} = \left(\frac{I^* + I}{2}\right)(C_1)\ (r)\left(\frac{1}{1+r}\right)$$

The derivation of a formula for Purchase Cost in each case simply involves determining the amount of Phosphate Rock purchased in each period, multiplying it by the cost of Phosphate Rock for that period and applying the Cost of Capital to discount the period cost to the present. As with Carrying Cost the present values of each period are then summed to determine the total present value for a particular strategy and consumption rate.

e.g. $u = 1370$ short tons/day
Third period (Mid 1978 to Mid 1979)

$$\text{Rock Purchased} = 365u$$

$$\text{Cost of Purchase} = C_3$$

$$\text{Present Value for Period} = 365u\left[\frac{1}{(1+r)^2}\right]$$

The Lost Production cost is determined by evaluating the following equality developed in Chapter 1.

$$(3) \quad LPC = \frac{365u}{Q} \int_{\mu + \frac{S}{u}}^{\infty} k\left[x - \left(\mu + \frac{S}{u}\right)\right] f(x)\, dx$$

The value for k was chosen to be \$50,000/day for all cases thus once again biasing the comparison in favour of the second strategy.

The integral was evaluated in the following manner:

1. A table of values for the function $f(x)$ was determined from Normal Tables[18] so that the probability of a trip exceeding the mean trip time by any daily increment could be determined. (See TABLE 18)

2. The expected cost for any runout situation was then determined by multiplying the daily cost by the runout probability and summing all the daily costs.

e.g. If $s = 30{,}000$ short tons
$u = 2{,}000$ short tons/day

No probability of runout exists until after the 15th day ($s/u = 15$).

$$\therefore \text{Expected Cost} = 80{,}000(.0024) + (2)(80{,}000)(.0016) + (3)(80{,}000)(.0011) + \text{- - - -} = \$2720/\text{trip}$$

The results of these calculations are summarized in TABLE 19. The Lost Production Cost can then be obtained by simply referring to TABLE 19.

e.g. $u = 2732$ short tons/day
Third period (Mid 1978 to Mid 1979)

I = 50,000 short tons
S = I − Q/2 = 36,000 short tons
Number of Days beyond mean trip time
beyond which a probability of runout exists
= S/u = 13

∴ Expected cost/Trip = $5888 (from TABLE 19)

$$\text{Number of Trips/Year} = \frac{365u}{Q} = 35.6$$

∴ Annual Cost = (5888)(35.6) = $210,000

The present value is then determined by applying the discount rate in the normal fashion.

Application of the above methodology thus led to the development of the following formulae:

1. Strategy 1

a) Carrying Cost

$$= Ir\left[C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2}\right]$$

b) Purchase Cost

$$= 365u\left[C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2}\right]$$

c) Lost Production Cost

= 0/ 0/ 0/ 290,000

2. Strategy 2

a) Plant Rate of 685 short tons/day (A)

(i) Carrying Cost

$$= \left(\frac{I^*+I}{2}\right)rC_1 + \left(\frac{2I^*-365u}{2}\right)r\frac{C_1}{1+r} + \left(\frac{I^*-365u+I}{2}\right)r\left[\frac{C_1}{(1+r)^2}\right]$$

(ii) Purchase Cost

$$= (I^* - I + 365u)\ C_1$$

(iii) Lost Production Cost

$= 0$

b) Plant Rate of 912 short tons/day (B)

(i) Carrying Cost

$$= \left(\frac{I^*+I}{2}\right) rC_1 + \left(\frac{2I^*-365u}{2}\right) r \frac{C_1}{1+r} + \left(\frac{I^*-365u+I}{2}\right) r \left(\frac{C_1}{(1+r)^{\frac{1}{4}}}\right) + Ir\left(\frac{C_3}{(1+r)^{\frac{9}{4}}}\right)$$

(ii) Purchase Cost

$$= (I^* - I + 365u)\ C_1 + 183u \left(\frac{C_3}{(1+r)^{\frac{9}{4}}}\right)$$

(iii) Lost Production Cost

$= 0$

c) Plant Rate of 1370 short tons/day (C)

(i) Carrying Cost

$$= \left(\frac{I^*+I}{2}\right) rC_1 + \left(\frac{I^*+I}{2}\right) r \left(\frac{C_1}{1+r}\right) + Ir\left(\frac{C_3}{(1+r)^2}\right)$$

(ii) Purchase Cost

$$= (I^* - I + 365u)\ C_1 + 365u \left(\frac{C_3}{(1+r)^2}\right)$$

(iii) Lost Production Cost

$= 0$

d) Plant Rate of 2732 short tons/day (D)

(i) Carrying Cost

$$= \left(\frac{I^*+I}{2}\right) rC_1 + \left(\frac{I^*+I}{2}\right) r \left(\frac{C_1}{(1+r)^{\frac{5}{4}}}\right) + Ir\left[\frac{C_2}{(1+r)^{\frac{5}{4}}} + \frac{C_3}{(1+r)^2}\right]$$

(ii) Purchase Cost

$$= (I^*-I+365u)\ C_1 + 183u \left(\frac{C_2}{(1+r)^{\frac{5}{4}}}\right) + 365u \left(\frac{C_3}{(1+r)^2}\right)$$

(iii) Lost Production Cost

$= 130{,}000$

The analysis in each case was carried out over a three year period commencing in mid 1976.

For the purpose of illustration the following variables were set at fixed values:

C_1 = \$10

I^* = 550,000 short tons

I = 50,000 short tons

r = 12%

Values for C_2 , C_3 and u were allowed to vary so that a plot of cost differential between strategy 1 and strategy 2 versus Phosphate Rock price increase ($C_2 - C_1$ or $C_3 - C_2$) at different plant consumption rates could be determined. The results are summarized in TABLE 10 and FIGURE 6.

To test the sensitivity of the results the Cost of Capital, r, was varied from 10 to 15% with all other parameters held constant. The results of this sensitivity analysis are summarized in FIGURE 7.

D. Summary and Conclusions

1. The static optimization model developed in Chapter 1 can be used with confidence if Phosphate Rock prices (FOB Beamer) are increasing at an annual rate of less than $10. The model is thus suitable for the foreseeable future since prices are predicted to increase by an annual amount of five to six dollars. (TABLE 9)

2. Two valid reasons exist for maintaining high inventory levels until June 30, 1977.

   a) Such a strategy improves Esso Chemical Canada's bargaining position with Phosphate Rock suppliers.

   b) The prohibitive penalty associated with the C.N.R. contract.

CHAPTER 3

ALTERNATIVE SOURCES OF FUTURE PHOSPHATE ROCK SUPPLIES

A. World Supply[19, 20]

Phosphorous is the twelfth most abundant element in the earth's crust. Estimated world reserves of Phosphate Rock total 140 billion short tons with 85% located in Morocco, U.S.A. and U.S.S.R. About 50 billion tons are recoverable using present methods - enough to supply the world demand for 500 years based on current world consumption rates. TABLE 11 summarizes total world reserves by area. TABLE 12 summarizes total world recoverable reserves by area. TABLE 13 summarizes predicted area production capabilities in 1978.

A brief description of potential suppliers follows:

1. Morocco

Morocco, based on its recoverable reserves and quality of Phosphate Rock is the most important Phosphate Rock producing area in the world today. Production capability is not high in comparison with recoverable reserves but expansion plans are in progress - mainly long term joint ventures with the U.S.S. R. Costs, F O B Beamer, Alberta are high in comparison to alternative comparable supplies. (+ $30/ short ton)

2. U.S.S.R.

Russia will not contribute to the expanding world markets since its own needs, and those of its satellite

countries, will exceed its productive capacity - thus its interest in a joint venture with the Moroccans.

3. Spanish Sahara

The controversy with Morocco over ownership continues and the outcome remains in doubt thus making security of supply questionable. As with Moroccan Phosphate Rock costs, F O B Beamer, are not competitive with alternative suppliers.

4. U.S.A.

Production capacity suitable to meet projected demand will be available in both the Southern and Western U.S.A. in the foreseeable future. The cost of Phosphate Rock from either source (F O B Beamer, Alberta) is less than comparable Phosphate Rock from anywhere else in the world. The security of supply from either source is, as well, better than from any other world source.

Phosphate Rock from the Western U.S.A. is not as good a quality as Phosphate Rock from Florida, U.S.A. but has both a cost and logistical advantage. A comparison of the two supplies will be the subject of the next section of this paper.

5. Other

All other Phosphate Rock sources are suitable for "spot" purchases only due to cost, security of supply or stage of development considerations.

6. Summary

All the information available points to the U.S.A. being the only logical supplier of Phosphate Rock to Western Canada in the foreseeable future. Adequate reserves exist to last for at least 25 - 30 years. Development plans indicate that supply will meet or exceed demand in the foreseeable future. Costs, F O B Beamer, are less than from any other world source. Security of supply is second to no other potential supplier. The only question left unresolved is the choice between Western and Florida Phosphate Rock and is the subject of the next section of this chapter.

B. Western versus Florida Phosphate Rock

TABLE 14 summarizes the predicted costs for Phosphate Rock from the two alternative suppliers (FOB Beamer, Alberta). An apparent incentive to switch to Western Phosphate Rock exists in the near term but a number of penalties exist with Western Phosphate Rock and must be assessed before any final decisions can be made.

Two positive reasons exist for changing to Western Rock. Firstly, the cost is less. Secondly, since the number of links in the supply chain are fewer the variance of the supply should be less thus allowing the maintenance of lower inventory levels.

A number of disadvantages are also associated with a change to Western rock. Firstly, Western rock is finer and thus wind losses will be greater. Secondly, plant maintenance costs with Western rock are higher due to the increased scaling tendencies of Western rock. Thirdly, phosphorous losses are higher with Western rock due to the poorer digestive characteristics. Fourthly, it is unclear, at this point, if prices for Western rock are on an equivalent basis with Florida rock and a penalty may thus exist there as well. Fifthly, filterability of Western rock crystals is only 60 - 70% of Florida rock filterability thus reducing effective maximum plant throughputs by 30 - 40%.

The balance of this paper is concerned with assessing the total impact of the penalties and incentives based on potential changes from the previously established optimum inventory if Western rock was used. The capacity penalty is ignored since it is not a real penalty

in a depressed market situation and since determination of the overall incentive exclusive of the capacity penalty establishes a basis for justification of plant changes to eliminate the capacity penalty. Costs and penalties are, of course, not rigidly established at this point and the following analysis is therefore only a first pass in the decision process.

1. <u>Inventory Considerations</u>

The standard deviation of trips from Vancouver to Beamer was 3.68 days, compared to a total trip standard deviation of 6.51 days. The trip from the Western U.S. to Beamer is naturally shorter than the Florida trip and includes fewer links in the transportation chain. However, it is doubtful that a standard deviation of 3.68 days can be achieved for trip times since three railways are involved and the potential for unit train movement is unknown at this time. For purposes of analysis a standard deviation of 4 days with a mean of 15 days is chosen in order to determine the minimum possible inventory level that could be maintained at Beamer. Completion of an identical analysis to the analysis carried out in CHAPTER 1 indicated that an inventory level at Beamer of 36,000 short tons of Western Rock would be comparable in risk to an inventory level of 50,000 short tons of Florida Rock at Beamer.

Therefore, it is indicated that the most optimistic assessment of improved risk with Western Rock allows an inventory reduction of only 14,000 short tons. However, the

need to build inventory to negate the impact of strikes would be avoided. Also, the demand for Western Rock will not be nearly as volatile as with the Florida Rock since offshore supply chains are not existent and will generally remain non-existent.

A strategy of inventory buildup with Florida Rock to 80,000 short tons in November/Decmeber and subsequent reduction to 50,000 short tons upon contract settlement actually yields an average inventory level at Beamer of 65,000 short tons. Thus further analysis is based on average permissable inventory levels of 65,000 short tons and 36,000 short tons with Florida and Western Rock respectively.

2. Maintenance Costs[21]

Experimentation with Western Phosphate Rock and discussions with other users of Western Rock indicate that maintenance costs will increase by $0.50 - $1.75 per short ton of Phosphate Rock.

3. Wind Losses[22]

Two per cent more Phosphate Rock is expected to be lost due to wind action with Western Rock. This equates to $0.80 per short ton of Phosphate Rock. However, losses are directly proportional to the amount of outside inventory and lower Western Rock inventory levels will offset this cost. The inventory change actually results in a credit of $0.08 per short ton of Phosphate Rock.

4. Phosphorous Process Losses[23]

Plant tests indicate that an increased loss of two per cent is anticipated. This equates to $0.80 per short ton of Phosphate Rock.

5. Carrying Cost Reduction

An inventory reduction of 29,000 short tons results in a carrying cost reduction of $0.30 per short ton of Phosphate Rock based on a Cost of Capital of 12% and Phosphate Rock cost of $44 per short ton.

In order to assess the impact of the various penalties over the next three years each penalty was escalated in an appropriate fashion. Maintenance costs were escalated at 8% per year. All other costs were escalated on the basis of changing Phosphate Rock prices. The total penalty escalation is summarized in TABLE 15. TABLE 16 summarizes the incentives that exist to change to Western Rock. Both tables are exclusive of rate considerations.

This analysis indicates that no incentive exists to change to Western Rock until 1979. If Western Rock was used in 1979 and 1980 and the plant was debottlenecked to 830,000 short tons per year cash flows of $2,042,000 in 1979 and $3,528,000 in 1980 above what would be achieved with Florida Rock would occur.

C. Summary and Conclusions

1. No apparent incentive exists to switch to Western Rock in 1977 or 1978.

2. Consideration should be given to securing Florida Phosphate Rock contracts for the immediate future with a view to securing Western Phosphate Rock contracts in the medium term. Also, consideration should be given to debottlenecking the plant to handle Western Rock at capacity by 1980.

The above conclusions are, of course, based on the best available cost data today. Further plant testing or a more detailed review of the costs of other users of Western Rock could change the maintenance and loss penalties. More critical, however, is the grade comparability. The standard deviation used for Western Rock trips was quite subjective and assumed that unit trains were not available. Much additional work is required to firm up the transportation mode and the attendant standard deviation for the travel time as the impact on costs is significant. However, the data is of a high enough quality to indicate that no great incentive exists to change to Western Rock in the short term unless lower prices can be negotiated for Western Rock.

CHAPTER 4

GENERAL CONSIDERATIONS

The analysis has indicated that a significant incentive does exist to maintain Inventory levels at more optimum levels. The data and discussion have indicated desirable levels that should be maintained in "normal" and "abnormal" times. The incentive and amount of effort required to continually monitor and regulate inventory levels dictate a definite resourcing committment is required. To date a good deal of effort has been invested in the day to day management of the various contracts, the scheduling of Phosphate Rock shipments and preparatory work for the preparation of new contracts in 1977 by the Toronto Supply Group. The work carried out has been excellent and the work load was obviously not small. If the proposals made earlier are worthy of additional effort then the Toronto Supply Group must determine if the additional effort can be readily handled by the existing force or if additional resourcing is required. Consideration should perhaps be given to computerization of the methods employed previously. Such an effort would potentially reduce the need for sustained additional resourcing.

Cost accountability for Inventory Carrying Costs are not clear or at least not explicit. It would be worthwhile to make them explicit in the cost statements of Agricultural Chemicals. This could presumably be done very simply by determining the Cost of Capital on a regular basis or even using a constant figure such as 12% and applying it to the average inventory level maintained at Beamer. i.e.

It is a true cost of operating and should be stewarded to by the personnel responsible for managing the Agricultural Chemicals Operations.

The maintenance of relatively large Inventory Levels at the present time is reasonable since contract negotiations are in progress and no guarantee exists that new contracts will be in place on June 30, 1977. Thus a high level of inventory does improve Esso Chemical Canada's bargaining position.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The purpose of this section is to summarize all the conclusions drawn in previous sections thus ensuring consistency and also to act as a comprehensive summary for the reader.

1. It is doubtful that an incentive exists to move away from Florida Phosphate Rock in the short term (1977 and 1978). Certainly with the exception of Western Phosphate Rock no competitive suppliers exist anywhere else in the world. Western Phosphate Rock is very worthy of consideration as a source of supply commencing in 1979. In order to make such a decision additional work is required in the following areas:

a) Better definition of costs associated with running Western Rock at Beamer.

b) A clear definition of the facilities required together with the Capital Investment which would allow plant capacity to be increased to a level such that plant rates would be equal with either Western Rock or Florida Rock.

c) Better definition of future costs of Western and Florida Phosphate Rock on a comparable Phosphorous content basis.

It is recognized that a lot of effort has been put into these areas in the last two years and is continuing in preparation for new contract negotiations. The comments are included here simply for reasons of completeness.

2. Building Inventory Levels as a result of anticipated future

price increases does not make economic sense unless the cost (FOB Beamer) is increasing by at least ten dollars per short ton annually. Predicted price increases over the foreseeable future thus do not dictate an inventory buildup policy as being appropriate. An increase of ten dollars per short ton of Phosphate Rock is a significant increase - it represents a 53% increase in the price of Florida Rock in 1977.

3. Under existing or similar contracts normal average inventory levels should be maintained in the 50,000 short ton range with a view to increasing and decreasing levels prior to and subsequent to, respectively, any contractual periods associated with dock workers at Tampa or Vancouver. The actual levels to which inventory should be increased during contractual periods is naturally based on judgment and the most current information from informed sources familiar with the potential for strike action. Historical information and a judgment as to the likely mood of any Federal Government would suggest that an increase to the 80,000 to 100,000 short ton range would be appropriate.

4. The management of Phosphate Rock Inventory has a high incentive associated with it ($500,000 per year) and therefore should be resourced properly and accountability established in a proper fashion.

Resourcing requires a review by the Toronto Supply Group to determine whether or not additional resourcing is appropriate in the short or long term.

Cost accountability is best handled by having the Carrying Cost appear in an explicit fashion in the cost statements of Agricultural Chemicals.

APPENDIX 1

CALCULATIONS OF MEANS AND STANDARD DEVIATIONS FOR TRIP DURATIONS

The data from TABLES 2, 3 and 4 was combined to determine total trip times for each of the nineteen trips. Formula used for the calculations were:[8, 9]

$$\mu = \frac{\sum_{i=1}^{19} x_i}{19}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{19} x_i^2 - 19\mu^2}{18}}$$

TABLE 5 summarizes the intermediate calculations. TABLE 6 summarizes the values of μ and σ calculated for each trip.

TABLE 1

SUMMARY OF CONTRACT TONNAGE OBLIGATIONS

CONTRACT	ECC OBLIGATIONS (SHORT TONS/YEAR)	SECOND PARTY OBLIGATIONS (SHORT TONS/YEAR)	PENALTIES PAYABLE BY ECC FOR TONNAGE BREACHES ($/SHORT TON)
MOBIL	- 50% (± 5%) OF REQUIREMENTS - NO MINIMUM	UP TO 400,000	-
SWIFT	- 50% (± 5%) OF REQUIREMENTS - NO MINIMUM	UP TO 400,000	-
STAR SHIPPING	-	UP TO 1,008,000	-
NEPTUNE TERMINALS	- 190,000 UNTIL TOTAL OF 2,850,000 SHORT TONS REACHED	AS REQUIRED BY OTHER CONTRACTS	N/A
CNR	- 51,000/MONTH -714,000	- UP TO 816,000 - UP TO 916,000 WITH NEGOTIATION	0.85

Source: Five Contracts described in Bibliography

TABLE 2

SHIP ARRIVAL TIMES AT TAMPA, FLORIDA

VOYAGE	PLANNED ARRIVAL DATE	ACTUAL ARRIVAL DATE	DIFFERENCE (DAYS)
100	Dec. 4/74	Jan. 2/75	-29
101	Dec. 18/74	Jan. 21/75	-34
102	Jan. 1/75	Jan. 19/75	-18
103	Jan. 15/75	Feb. 6/75	-22
104	Jan. 29/75	Feb. 20/75	-22
105	Feb. 5/75	March 3/75	-26
106	Feb. 19/75	March 7/75	-16
107	March 7/75	April 5/75	-29
108	March 19/75	April 14/75	-26
109	April 2/75	April 22/75	-20
110	May 7/75	May 26/75	-19
111	May 21/75	June 3/75	-13
112	June 4/75	June 29/75	-25
113	July 15/75	July 2/75	+13
114	Aug. 10/75	July 29/75	+12
115	Aug. 25/75	Aug. 13/75	+12
116	Sept. 5/75	Sept. 10/75	-5
117	Oct. 15/75	Oct. 11/75	+4
118	Nov. 10/75	Nov. 17/75	-7
119	Nov. 20/75	Nov. 29/75	-9
120	Dec. 20/75	Dec. 23/75	-8
121	Dec. 31/75	Dec. 31/75	0
122	Jan. 13/76	Jan. 8/76	+5
123	Jan. 28/76	Feb. 2/76	-5
124	Feb. 9/76	Feb. 5/76	+4
125	Feb. 14/76	Feb. 19/76	-5
126	Feb. 26/76	Feb. 24/76	+2
127	March 15/76	March 18/76	-3
128	March 23/76	March 30/76	-7
129	April 2/76	April 4/76	-2
130	April 16/76	April 17/76	-1
132	Oct. 1/76	Oct. 4/76	-3

Source: Esso Chemical Canada Plant Records

TABLE 3

ELAPSED TIME - LOADING COMMENCEMENT AT TAMPA TO UNLOADING COMPLETION AT VANCOUVER

VOYAGE	DURATION (DAYS)
100	33
101	32
102	29
103	45
104	38
105	39
106	26
107	22
108	26
109	22
110	28
111	27
112	20
113	23
114	23
115	25
116	22
117	23
118	22
119	24
120	20
121	23
122	31
123	23
124	24
125	20
126	21
127	22
128	24
129	28
130	26
131	19

Source: Esso Chemical Canada Plant Records

TABLE 4

ELAPSED TIME - LOADING COMMENCEMENT AT VANCOUVER TO UNLOADING COMPLETION AT BEAMER

PSEUDO VOYAGE	DURATION (DAYS)
100	20
101	9
102	8
103	13
104	18
105	18
106	15
107	16
108	13
109	11
110	11
111	15
112	23
113	17
114	17
115	17
116	19
117	20
118	16
119	12
120	11
121	15
122	15
123	13
124	12
125	12
126	12
127	12
128	12
129	13
130	22
131	23

Source: Esso Chemical Canada Plant Records

TABLE 5

INTERMEDIATE DATA FOR MEAN AND STANDARD DEVIATION CALCULATIONS

TABLE 2		TABLE 3		TABLE 4		TOTAL	
x_i	x_i^2	x_i	x_i^2	x_i	x_i^2	x_i	x_i^2
-13	169	23	529	17	289	27	729
-12	144	23	529	17	289	28	784
-12	144	25	625	17	289	30	900
5	25	22	484	19	361	46	2116
-4	16	23	529	20	400	39	1521
7	49	22	484	16	256	45	2025
9	81	24	576	12	144	45	2025
8	64	20	400	11	121	39	1521
0	0	23	529	15	225	38	1444
-5	25	31	961	15	225	41	1681
5	25	23	529	13	169	41	1681
-4	16	24	576	12	144	32	1024
5	25	20	400	12	144	37	1369
-2	4	21	441	12	144	31	961
3	9	22	484	12	144	37	1369
7	49	24	576	12	144	43	1849
2	4	28	784	13	169	43	1849
1	1	26	676	22	484	49	2401
3	9	19	361	23	529	45	2025
3	859	443	10473	290	4670	736	29274

TABLE 6

MEANS AND STANDARD DEVIATIONS FOR TRIP DURATIONS

STATISTIC	TAMPA ANNUAL (DAYS)	TAMPA-VANCOUVER (DAYS)	VANCOUVER-BEAMER (DAYS)	TAMPA-BEAMER (DAYS)
Mean (μ)	0.16	23.32	15.26	38.74
STANDARD DEVIATION (σ)	6.91	2.83	3.68	6.51

TABLE 7

GOODNESS OF FIT TEST DATA

(1) TRIP DURATION (DAYS) x	(2) ACTUAL TRIP FREQ. f_a	(3) NORMAL DEVIATE $z = \frac{x - 33.74}{6.51}$	(4) AREA TO LEFT OF x	(5) AREA OF CLASS INTERVAL	(6) EXPECTED FREQ. (5) × 19 f_e	(7) $f_a - f_e$	(8) $(f_a - f_e)^2$	(9) $\frac{(f_a - f_e)^2}{f_e}$
30	2	-1.34	.0901	.0901	1.72	0.28	0.08	0.047
30-34	3	- .73	.2327	.1426	2.71	0.29	0.08	0.030
35-39	5	.04	.5160	.2833	5.38	-0.38	0.14	0.026
40-44	4	.81	.7910	.2750	5.23	-1.23	1.51	0.289
45-49	5	1.58	.9429	.1519	2.89	2.11	4.45	1.540
>49	0	∞	1.0000	.0571	1.08	-1.08	1.17	1.083
	19			1.0000	19.00		$\chi^2 =$	3.015

DEGREES OF FREEDOM = 6-1 = 5

TABLE 8

PROBABILITY DATA FOR TRIP DURATIONS FROM TAMPA TO BEAMER

PROBABILITY THAT ACTUAL TRIP TIME IS LESS THAN POSITIVE DEVIATION VALUE INDICATED	POSITIVE DEVIATION FROM MEAN TRIP TIME			
	TAMPA ARRIVAL (DAYS)	TAMPA TO VANCOUVER (DAYS)	VANCOUVER TO BEAMER (DAYS)	TAMPA TO BEAMER (DAYS)
0.84	6.91	2.83	3.68	6.51
0.9772	13.82	5.66	7.36	13.02
0.9987	20.73	8.49	11.04	19.53
0.99997	27.64	11.32	14.72	26.04

TABLE 9

PREDICTED PHOSPHATE ROCK PRICES TO 1985

TIME PERIOD	COST FOB MINE ($/ST)	LOGISTICS COST ($/ST)	TOTAL COST ($/ST)
MID 76-JUNE 30/77	13.50	25	38.50
BALANCE OF 1977	16	26	42
1978	20	28	48
1979	22	31	53
1980	26	33	59
1985	34	45	79

Source: Hryciuk, Ray B.; Esso Chemical Canada Supply Group

TABLE 10

NET PRESENT COST OF STRATEGY 1 LESS STRATEGY 2 AT VARIOUS PHOSPHATE ROCK PRICES AND PLANT CONSUMPTION RATES

ANNUAL PHOSPHATE ROCK PRICE INCREASE (%) \ PLANT RATE (SHORT TON/DAY)	685	912	1370	2732
5	(3,200,000)	(2,900,000)	(2,150,000)	(2,100,000)
10	(11,000)	(65,000)	106,000	18,000
20	6,360,000	5,690,000	4,630,000	4,740,000
30	12,720,000	11,430,000	9,140,000	9,600,000

TABLE 11

WORLD RESERVES OF PHOSPHATE ROCK

AREA	RESERVES (MMMST)
Morocco	60.0
U.S.A.	50.0
U.S.S.R.	11.0
Africa (Excluding Morocco)	17.0
South America	0.3
Europe & Near East (Excluding USSR)	1.0
Asia	1.0
Oceania	3.0
Total	143.3

Source: Hryciuk, Ray B.; *Esso Chemical Canada World Phosphate Rock Study - Phase I: Reserves and Supply/Demand Balances*; October, 1974; p. 2

TABLE 12

RECOVERABLE WORLD RESERVES OF PHOSPHATE ROCK

AREA	RECOVERABLE RESERVES (MMMST)
Morocco	40.0
U.S.A.	5.6
U.S.S.R.	3.5
Africa (Excluding Morocco)	2.5
South America	0.1
Europe & Near East (Excluding USSR)	0.2
Asia	0.2
Oceania	0.4
Total	52.5

Source: Hryciuk, Ray B.; Esso Chemical Canada World Phosphate Rock Study - Phase I: Reserves and Supply/Demand Balances; October, 1974; p. 3

TABLE 13

WORLD PHOSPHATE ROCK PRODUCTION CAPACITY (1978)

AREA	PRODUCTION CAPACITY (MMST)
U.S.A.	
Southern	60.0
Western	9.5
U.S.S.R.	25.0
Morocco	26.0
Tunisia	5.0
Ocean, Nauru	3.0
China	3.5
Togo	2.6
Senegal	2.0
Christmas Island	1.5
South Africa	2.8
Spanish Sahara	6.0
Egypt	1.5
Algeria	1.5
Jordon	2.5
Israel	1.0
Syria	1.0
Australia	3.0
Peru	1.0
Other	4.1
Total	162.5

Source: Wayne Thomas Inc.; The "Florida" Phosphate Industry Domestic and World Markets; April 9, 1975; p. 34

TABLE 14.

FUTURE PHOSPHATE ROCK COSTS (FOB BEAMER)

TIME	FLORIDA ($/SHORT TON)	WESTERN ($/SHORT TON)
1977	44	44
1978	47	46
1979	56	51
1980	63	56
1985	79	63

Source: Hryciuk, Ray B.; Esso Chemical Canada Supply Group

TABLE 15

PENALTIES ASSOCIATED WITH USING WESTERN ROCK

PENALTY AREA	COST OF PENALTY ($/SHORT TON)			
	1977	1978	1979	1980
MAINTENANCE	1.75	1.89	2.04	2.20
PHOSPHOROUS LOSS	0.80	0.88	0.94	1.04
WIND LOSSES	(0.08)	(0.09)	(0.09)	(0.10)
CARRYING COST	(0.30)	(0.33)	(0.35)	(0.39)
TOTALS	2.17	2.35	2.54	2.75

TABLE 16

INCENTIVE TO CHANGE TO WESTERN ROCK

ITEM	COST ($/SHORT TON)			
	1977	1978	1979	1980
Western	44	46	51	56
Florida	44	47	56	63
Penalty	2.17	2.35	2.54	2.75
INCENTIVE	(2.17)	(1.35)	2.46	4.25

TABLE 17

HISTORICAL INVENTORY LEVELS

DATE (MONTH/YEAR)	NEPTUNE (SHORT TONS)	BEAMER (SHORT TONS)
1/75	7500	113350
2/75	38000	104337
3/75	38000	77300
4/75	38501	106750
5/75	39116	120163
6/75	26100	115000
7/75	28384	230900
8/75	27000	213000
9/75	14882	194823
10/75	23490	176718
11/75	25764	160581
12/75	18000	140000
1/76	33000	120000
2/76	41000	114000
3/76	18500	171000
4/76	19000	185000
5/76	38000	210000
6/76	21000	214000
7/76	21293	162382
8/76	24904	152570
9/76	622	168285
10/76	20846	153692
11/76	48166	151865
12/76	23542	154133

SOURCE: ESSO CHEMICAL CANADA PLANT RECORDS

TABLE 18

PROBABILITY THAT TRIP TIME EXCEEDS THE MEAN

TRIP TIME BY ANY PARTICULAR VALUE

DAY BEYOND THE MEAN	PROBABILITY
1	.0596
2	.0621
3	.0555
4	.0519
5	.0503
6	.0418
7	.0365
8	.0330
9	.0255
10	.0220
11	.0163
12	.0126
13	.0101
14	.0070
15	.0051
16	.0038
17	.0024
18	.0016
19	.0011
20	.0007
21	.0004
22	.0003
23	.0002
24	.0001
25	.00004
26	.00003

TABLE 19

EXPECTED COST OF RUNNING OUT OF PHOSPHATE ROCK

NO. OF DAYS WITHOUT RUNOUT POSSIBILITY (s/u)	LOSS ($/DAY)		
	10,000	50,000	80,000
1	23,647	118,235	189,176
2	19,232	96,160	153,856
3	15,450	77,250	123,600
4	12,222	61,110	97,776
5	9,503	47,515	76,024
6	7,318	36,490	58,384
7	5,510	27,550	44,080
8	4,077	20,385	32,616
9	2,984	14,920	23,872
10	2,147	10,735	17,176
11	1,530	7,650	12,240
12	1,064	5,320	8,512
13	736	3,680	5,888
14	508	2,540	4,064
15	340	1,700	2,720
16	234	1,170	1,862
17	155	775	1,240
18	99	495	792
19	61	305	488
20	29	145	232
21	19	95	152

FIGURE 1

PHOSPHATE ROCK SUPPLY CHAIN

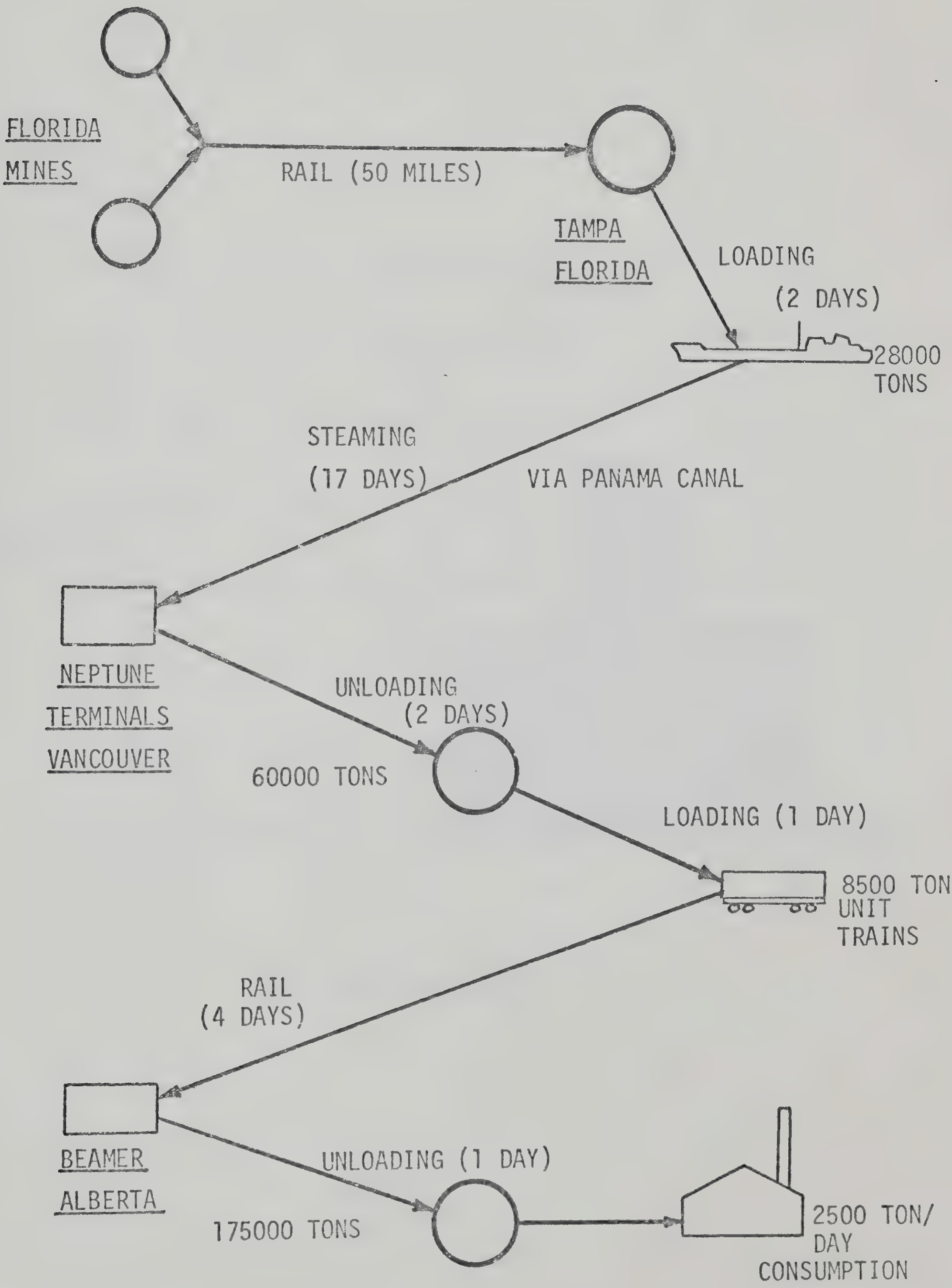


FIGURE 2

HISTOGRAM FOR TRIP DURATION FROM TAMPA TO BEAMER

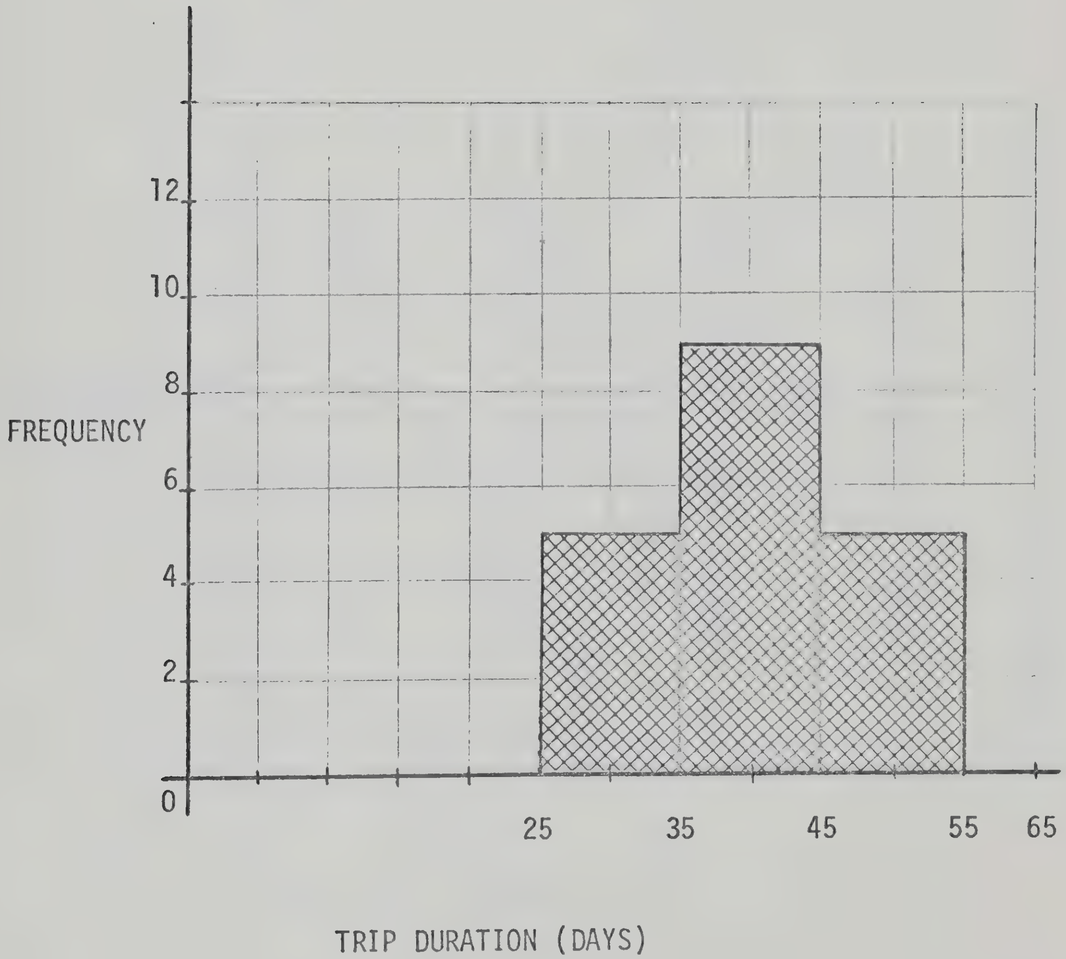


FIGURE 3

INVENTORY MODEL ILLUSTRATION

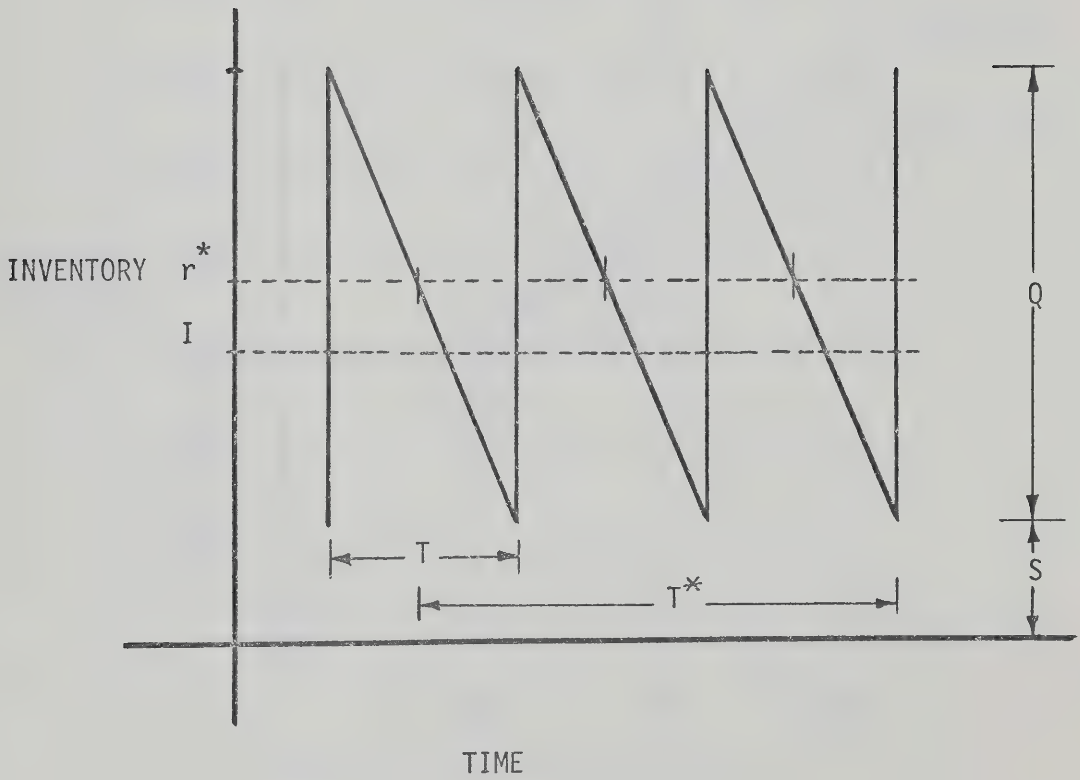


FIGURE 4

OPTIMUM INVENTOTY LEVELS VS DEMAND FOR PHOSPHATE ROCK

AT VARIOUS DAILY PENALTIES

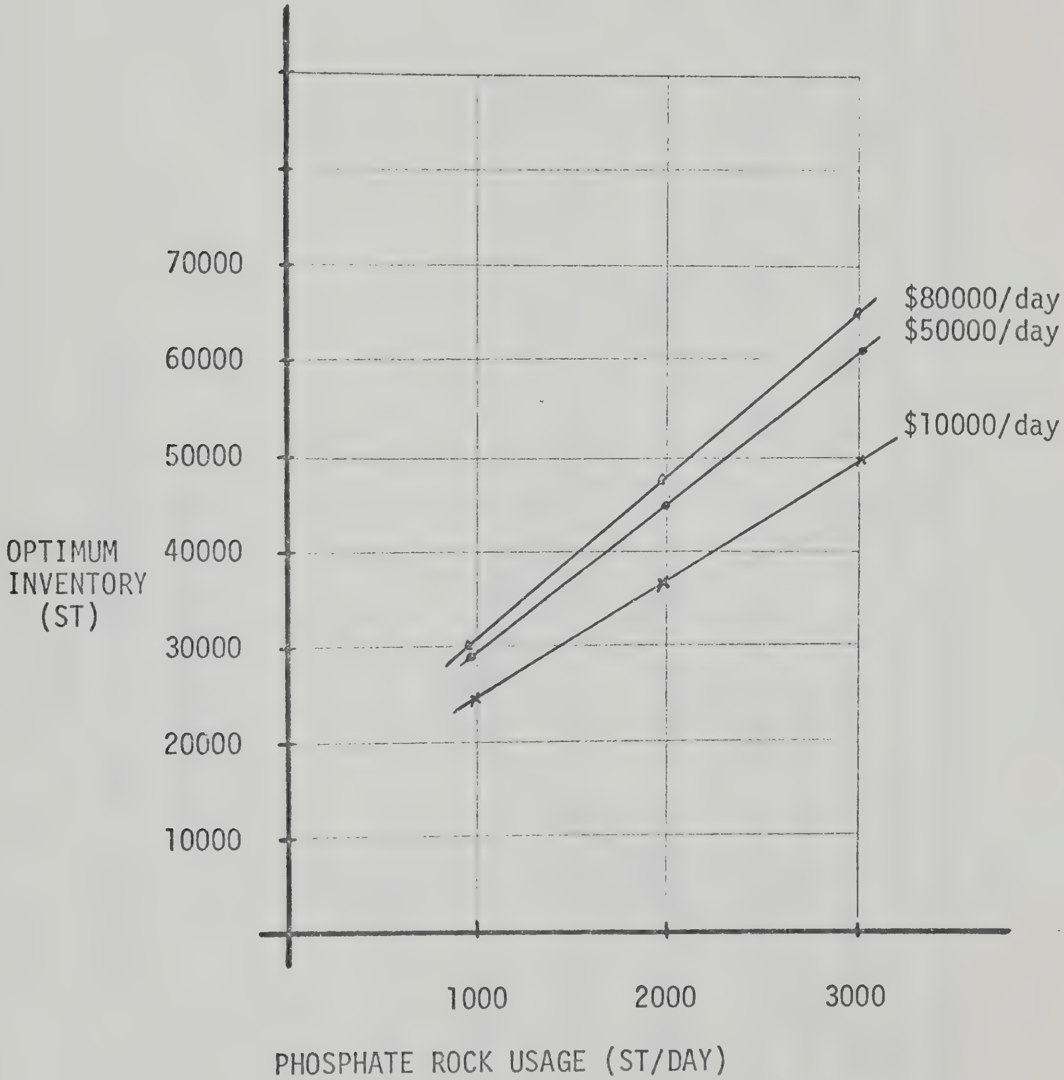


FIGURE 5

INVENTORY LEVELS OF PHOSPHATE ROCK UNDER

ALTERNATIVE STRATEGIES

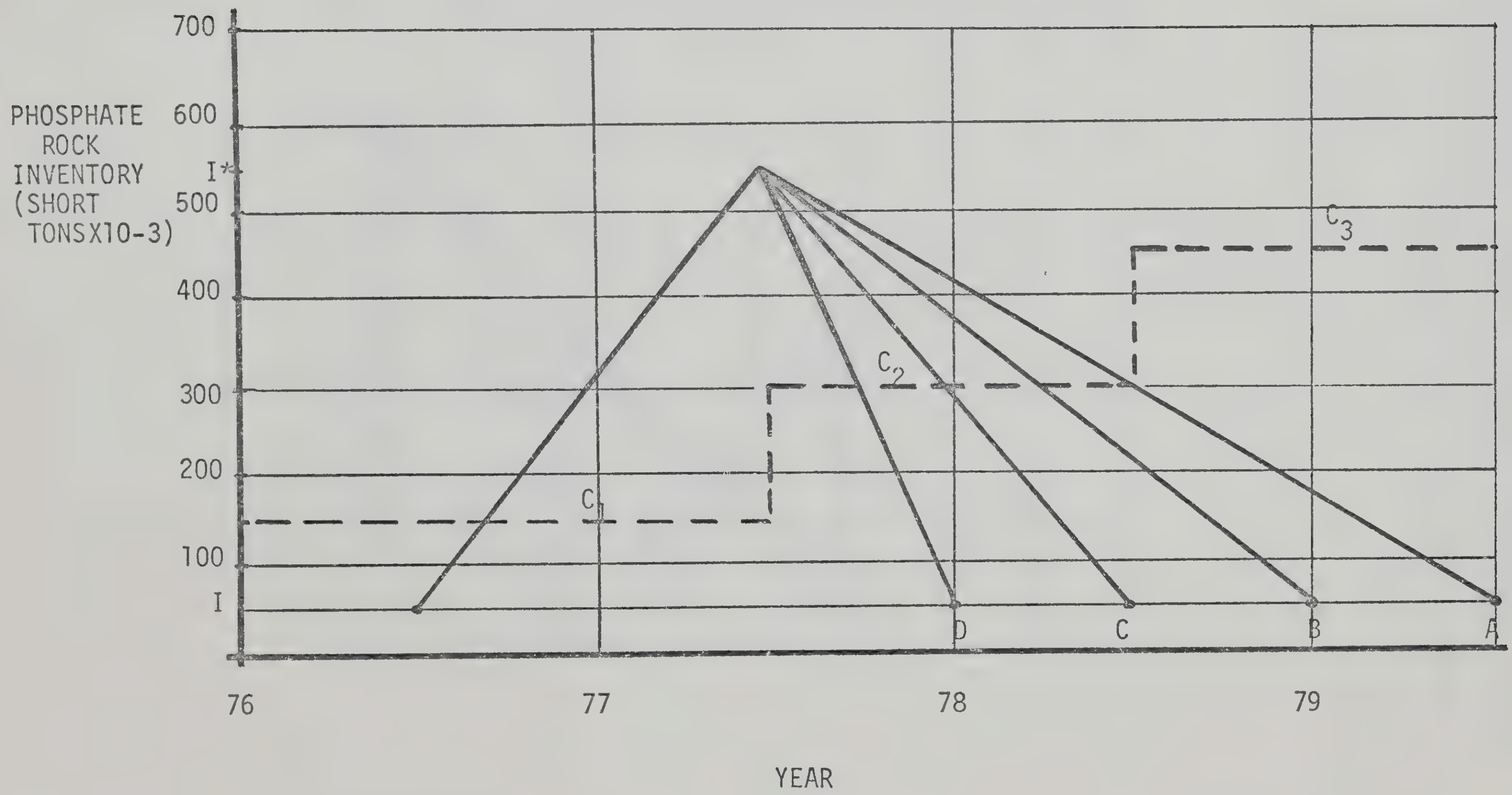


FIGURE 6

NET PRESENT COST OF STRATEGY 1 LESS STRATEGY 2 VS ANNUAL INCREASE IN PHOSPHATE ROCK PRICES AT VARIOUS PLANT CONSUMPTION RATES

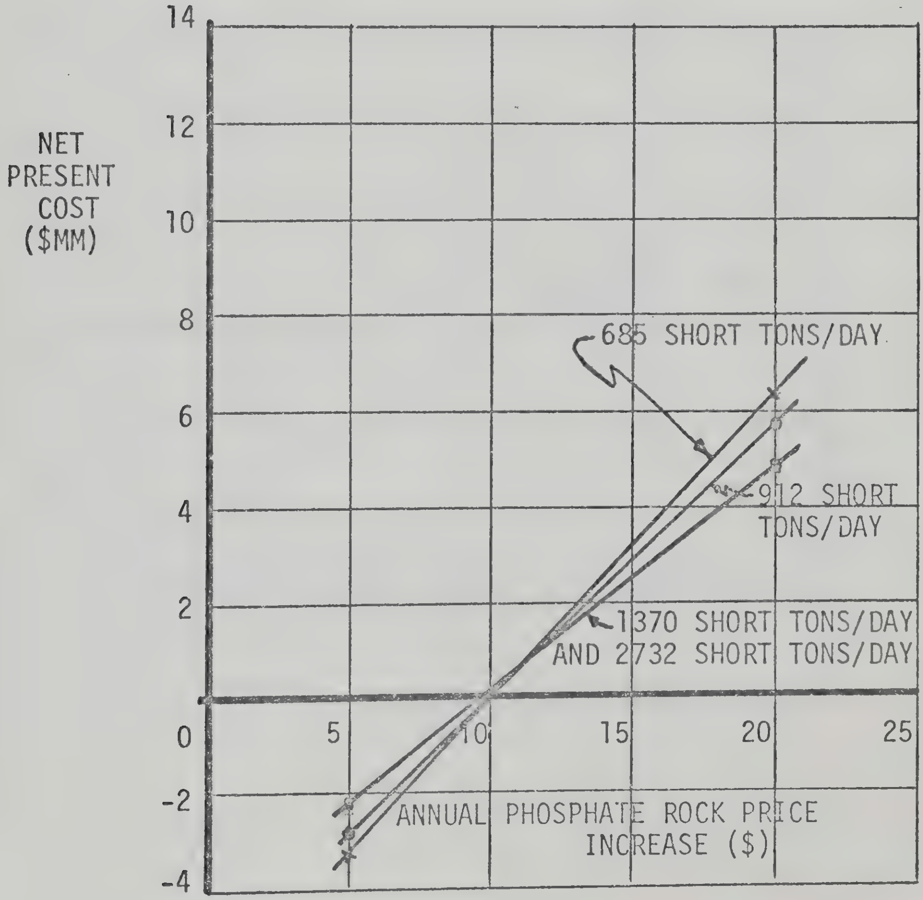
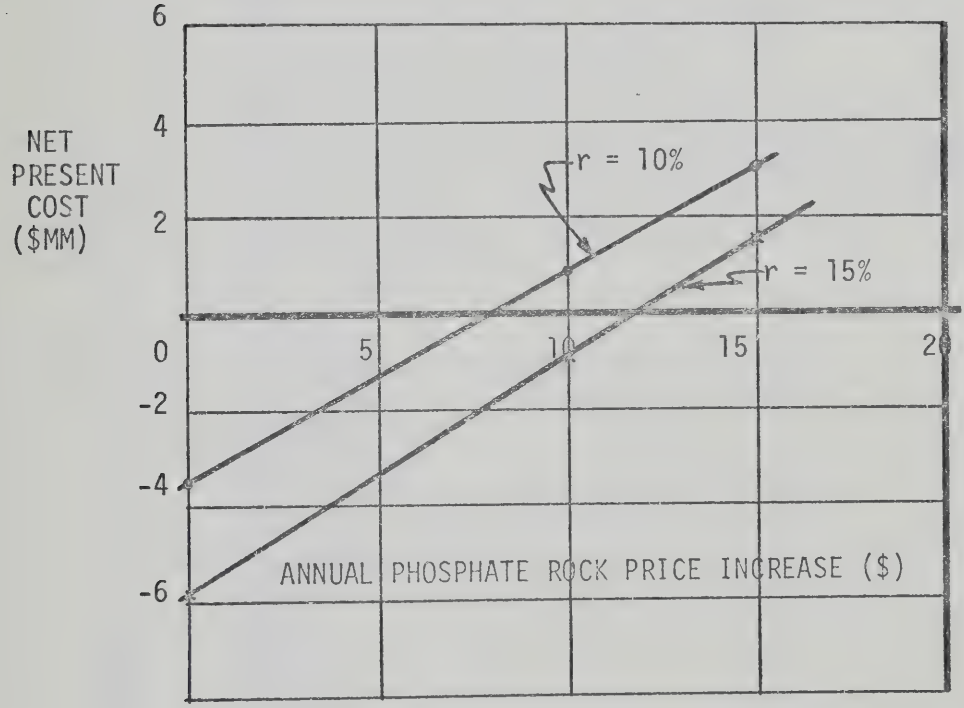


FIGURE 7

SENSITIVITY OF FIGURE 6 GRAPHS TO CHANGES

IN THE COST OF CAPITAL



GLOSSARY

μ	--	Mean (Days)
σ	--	Standard Deviation (Days)
χ	--	Goodness of Fit Test Statistic
x	--	Travel Time (Days)
fa	--	Actual Trip Frequency
fe	--	Expected Trip Frequency
z	--	Standard Normal Deviate
α	--	Level of Significance
r*	--	Inventory Reorder Level (Short Tons)
I	--	Average Inventory Level (Short Tons)
Q	--	Vessel Capacity (Short Tons)
T	--	Mean Time Between Vessel Arrivals (Days)
T*	--	Mean Time for Cargo Movement from Tampa to Beamer (Days)
S	--	Planned Minimum Safe Inventory Level (Short Tons)
CC	--	Carrying Cost of Inventory ($)
C_o	--	Phosphate Rock Cost - FOB Beamer ($)
r	--	Cost of Capital
LPC	--	Lost Production Cost
∞	--	Infinity
k	--	Daily Cost of Plant Shutdowns ($/Day)
$f(x)$	--	Probability of Cargo Arriving on Day x
$\int$	--	Integral Sign
d	--	Differential
F	--	Cumulative Normal Distribution Function
u	--	Plant Production Rate (Short Tons/Day)
ST	--	Short Tons
I*	--	Inventory Buildup Level (Short Tons)
MM	--	Million
MMM	--	Billion
FOB	--	Freight on Board
Σ	--	Summation

FOOTNOTES

[1]Mobil Chemical - Esso Chemical Canada Phosphate Rock Sales Contract, April 4, 1972.

[2]Swift Agricultural Chemicals - Esso Chemical Canada Phosphate Rock Sales Contract, March 21, 1972.

[3]Contract of Affreightment between Star Shipping A/S and Imperial Oil Limited, July 1, 1975.

[4]Phosphate Rock Terminalling Agreement between Imperial Oil Limited and Neptune Phosphate Terminals Ltd., November 1, 1967.

[5]Agreement between Imperial Oil Enterprises Ltd. and Canadian National Railway Company for Phosphate Rock Movement between Neptune Terminals Limited and Beamer, Alberta, November 11, 1974.

[6]Lapin, Laurence L.; Statistics for Modern Business Decisions; Harcourt Brace Jovanovich, Inc., 1973; p. 391-6

[7]Ibid., p. 174-5

[8]Ibid., p. 40

[9]Ibid., p. 57

[10]Hadley, G.; and Whitin, T. M.; Analysis of Inventory Systems; Prentice - Hall Inc., Englewood Cliffs, N. J., 1963; p. 1 - 25

[11]Johnson, Robert W.; Financial Management; Allyn and Bacon Inc., Boston, Fourth Edition, 1972; p. 243

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[13]Ibid., p. 345

[14]Lapin, Laurence L.; Statistics for Modern Business Decisions; Harcourt Brace Jovanovich, Inc., 1973; p. 162

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[19] Hryciuk, Ray B.; Esso Chemical Canada World Phosphate Rock Study - Phase I: Reserves and Supply/Demand Balances; October, 1974; p. 1-14

[20] Wayne Thomas, Inc.; The "Florida" Phosphate Industry Domestic and World Markets; April 9, 1975; p. 33-42

[21] Vukadinovic, M.; Western Phosphate Rock Evaluation; April 11, 1975; p. 4

[22] Ibid., p. 3

[23] Ibid., p. 3

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Swift Agricultural Chemicals - Esso Chemical Canada Phosphate Rock Sales Contract, March 21, 1972

Contract of Affreightment between Star Shipping A/S and Imperial Oil Limited, July 1, 1975

Phosphate Rock Terminalling Agreement between Imperial Oil Limited and Neptune Phosphate Terminals Ltd., November 1, 1967

Agreement between Imperial Oil Enterprises Ltd. and Canadian National Railway Company for Phosphate Rock Movement between Neptune Terminals Limited and Beamer, Alberta, November 11, 1974